Web appendix for:
Invoicing Currency and Financial Hedging

Victor Lyonnet[1], Julien Martin[2], and Isabelle Mejean[3]

[1] *The Ohio State University*
[1] *Université du Québec à Montréal, CREST, and CEPR*
[2] *CREST-Ecole polytechnique, and CEPR*

November 27, 2020

A Additional theoretical results

A.1 Proof of prosition 3.1

Recall that

$$\pi^{PCP}(S) = p^{PCP} D\left(\frac{p^{PCP}}{S}\right) - C\left[D\left(\frac{p^{PCP}}{S}\right), w(S)\right].$$

The first derivative of $\pi^{PCP}(S)$ with respect to S writes

$$\frac{d\pi^{PCP}(S)}{dS} = \eta D(.) \frac{p^{PCP} - mc}{S} - \frac{\partial C(.)}{\partial w(.)} \frac{\partial w(.)}{\partial S},$$

where $\eta \equiv -\frac{d \ln D(p^*)}{d \ln p^*}$, $mc \equiv \frac{\partial C(.)}{\partial D(.)}$ and we have used $\frac{dp^{PCP}}{dS} = 0$ in a one-period-ahead sticky-price setting. As in Burstein & Gopinath (2014), we allow the marginal cost of production to depend on the quantity produced as well as on the exchange rate: $mc = mc\left(D\left(\frac{p^{PCP}}{S}\right), S\right)$, where the exchange rate modifies the marginal cost of production insofar as some variable costs of production incurred by the exporting firm are local to the importing country. To simplify, we assume $\frac{\partial^2 w(.)}{\partial S^2} = 0$, that is, that $w(S)$ is linear in S. Under this assumption, the second derivative of $\pi^{PCP}(S)$ with respect to S writes:

$$\begin{aligned}\frac{d^2\pi^{PCP}(S)}{dS^2} =& \frac{d\eta}{dS}D(.)\frac{p^{PCP}-mc(.)}{S} \\ &+\eta\frac{dD(.)}{dp^{PCP}/S}\frac{dp^{PCP}/S}{dS}\frac{p^{PCP}-mc(.)}{S} \\ &-\eta D(.)\frac{p^{PCP}-mc(.)}{S^2} \\ &-\eta D(.)\frac{1}{S}\frac{dmc(.)}{dS}.\end{aligned}$$

with

$$\begin{aligned}\frac{dmc}{dS} =& \frac{\partial mc(.)}{\partial D(.)}\frac{dD(.)}{dS}+\frac{\partial mc(.)}{\partial S} \\ =& \frac{mc(.)}{S}\left(\eta mc_q + mc_S\right),\end{aligned}$$

where $mc_q \equiv \frac{\partial \ln mc(.)}{\partial \ln D(.)}$ is the elasticity of the marginal cost with respect to output and $mc_S \equiv \frac{\partial \ln mc(.)}{\partial \ln S}$ is the partial elasticity of the marginal cost with respect to the exchange rate. We finally obtain:

$$\frac{d^2\pi^{PCP}}{dS^2} = \eta D(.)\frac{p^{PCP}-mc(.)}{S^2}\left(-\frac{d\ln\eta}{d\ln\frac{p^{PCP}}{S}}+\eta-1-\frac{mc(.)}{p^{PCP}-mc(.)}\left(\eta mc_q+mc_S\right)\right),$$

and the concavity (convexity) of π^{PCP} with respect to the exchange rate S depends on the term within the parenthesis as given in proposition 3.1. QED.

A.2 Special case in the absence of hedging: Oligopolistic Competition

As discussed in the existing literature (Auer & Schoenle 2016, Amiti et al. 2014), the relationship between exchange rate pass-through / invoicing currency choices and firm size is non-linear under oligopolistic competition. We now show that our general model encompasses this situation. Following Auer & Schoenle (2016), we assume that preferences display nested CES with an upper layer in which consumers substitute across goods and/or across source countries at the elasticity σ and a lower layer in which consumers substitute across varieties produced by individual firms at the rate ρ. As is standard in this literature, we assume $1 < \sigma < \rho$. In this set-up, the demand addressed to a firm f producing a good g displays a constant elasticity:

$$q_g(f) = \left(\frac{p_g(f)}{P_g}\right)^{-\rho} Q_g$$

with P_g and Q_g respectively denoting the price index and real consumption addressed to producers of good g. Under CES, $Q_g = \left(\frac{P_g}{P}\right)^{-\sigma} Q$ with P and Q the aggregate price index and aggregate real consumption, respectively.

At the lower level, a finite number of non-atomistic firms are assumed to compete in quantities.[1] Under this assumption, the perceived elasticity of demand is decreasing in the firm's market share:

$$\eta_g(f) = \left[\frac{1}{\rho}(1 - s_g(f)) + \frac{1}{\sigma} s_g(f)\right]^{-1}$$

with $s_g(f) \equiv \frac{p_g(f) q_g(f)}{P_g Q_g} = \left(\frac{p_g(f)}{P_g}\right)^{1-\rho}$.

Based on these assumptions, one can rewrite condition (1) and derive the parametric conditions for the non-linearity. We do this here in the case in which the technology displays constant returns to scale and the marginal cost is independent of exchange rates ($mc_q = mc_S = 0$). Under these assumptions, LCP is optimal if:

$$\begin{aligned}
& \eta_g(f) - 1 - \frac{d \ln \eta_g(f)}{d \ln p_g(f)} < 0 \\
\Leftrightarrow \quad & \eta_g(f) - 1 - \frac{(\rho - \sigma)(\rho - 1)}{\sigma \rho} \eta_g(f) s_g(f)(1 - s_g(f)) < 0 \\
\Leftrightarrow \quad & (\rho - \sigma)(\rho - 1) s_g(f)^2 - \rho(\rho - \sigma) s_g(f) + \sigma(\rho - 1) < 0
\end{aligned}$$

One can then derive optimal invoicing choices as a function of the firm's market share given the roots of the quadratic equation:

$$s_1 = \frac{\rho(\rho - \sigma) - \sqrt{\Delta}}{2(\rho - \sigma)(\rho - 1)}, \quad s_2 = \frac{\rho(\rho - \sigma) + \sqrt{\Delta}}{2(\rho - \sigma)(\rho - 1)}$$

where $\Delta \equiv (\rho - \sigma)[\rho^2(\rho - \sigma) - 4\sigma(\rho - 1)^2]$

There are two regimes depending on the value of the ρ parameter. If the elasticity of substitution between firms is low enough (namely, if $\rho \leq 2$), the relationship between invoicing and size is linear, with small firms (such

[1] As shown in Auer & Schoenle (2016), qualitative results are robust to assuming firms to compete in prices.

as $s_g(f) \leq s_1$) pricing in PCP whereas large firms choose LCP. Instead, if $\rho > 2$, we find a non-linear relationship between firm size and invoicing currency choices: Firms with intermediate market shares ($s_1 < s_g(f) < s_2$) price in LCP while both small and large firms price in their own currency.

A.3 Hedging conditional on invoicing

In this sub-section, we derive the conditions under which the firm chooses to hedge, considering the two possible invoicing strategies sequentially.

LCP case. From the firm's program, we can show the firm chooses HLCP over LCP whenever

$$\mathbb{E}\left[u\left(\pi^{HLCP}(S)\right)\right] - \mathbb{E}\left[u\left(\pi^{LCP}(S)\right)\right] > 0.$$

From lemma 3.2, we know that, conditional on hedging, the firm hedges fully. Therefore, conditional on hedging, profits are certain ex ante:

$$\mathbb{E}\left[u\left(\pi^{HLCP}(S)\right)\right] = u\left(\pi^{HLCP}(\mathbb{E}[S])\right).$$

The first-order conditions of expected utility maximization with respect to prices and the hedging quantity are

$$\mathbb{E}\left[\frac{du(.)}{d\pi(.)}\left(S\left(p^{*,j}\frac{dD(.)}{dp^{*,j}} + D(.)\right) - mc\frac{dD(.)}{dp^{*,j}}\right)\right] = 0 \tag{A.1}$$

$$\mathbb{E}\left[\frac{du(.)}{d\pi(.)}\left(-S + f\right)\right] = 0, \tag{A.2}$$

where $j \in \{LCP, HLCP\}$. Rearranging and substituting (A.2) into (A.1) implies:

$$f\left(p^{*,j}\frac{dD(.)}{dp^{*,j}} + D(.)\right) = mc\frac{dD(.)}{dp^{*,j}}. \tag{A.3}$$

Condition (A.3) is independent of both the shape of the utility function and the stochastic properties of the exchange rate. This independence is a version of the "separation theorem" result that exchange rate uncertainty does not influence prices or traded quantities. We then write:

$$u\left(\pi^{HLCP}(\mathbb{E}[S])\right) = u\left(\pi^{LCP}\left(\mathbb{E}[S]\right) - F\right).$$

We approximate $u\left(\pi^{LCP}\left(\mathbb{E}(S)\right)-F\right) \simeq u\left(\pi^{LCP}\left(\mathbb{E}(S)\right)\right) - \frac{du(\pi^{LCP}(\mathbb{E}[S]))}{d\pi^{LCP}(\mathbb{E}[S])}F$. Inequality (2) obtains. A firm chooses HLCP over LCP if and only if:

$$u\left[\pi^{LCP}(\mathbb{E}[S])\right] - \mathbb{E}\left[u\left(\pi^{LCP}(S)\right)\right] > \frac{du\left(\pi^{LCP}(\mathbb{E}[S])\right)}{d\pi^{LCP}(\mathbb{E}[S])}F$$

PCP case. As before, a PCP firm chooses HPCP whenever

$$\mathbb{E}\left[u\left(\pi^{HPCP}(S)\right)\right] - \mathbb{E}\left[u\left(\pi^{PCP}(S)\right)\right] > 0.$$

Again, using lemma 3.2, we know that, conditional on hedging, the firm hedges fully. However, in contrast to the LCP case, expected utility from HPCP profits is not certain ex-ante:

$$\mathbb{E}\left[u\left(\pi^{HPCP}(S)\right)\right] = u\left(\pi^{HPCP}(\mathbb{E}[S])\right) - \Delta(S),$$

where $\Delta(S)$ is higher the more risk averse the firm's manager, and the sign of $\Delta(S)$ depends on condition (1). $\Delta(S) = 0$ if PCP profits are linear in the exchange rate.[2] If PCP profit is concave in the exchange rate (condition (1) is satisfied), $\Delta(S) > 0$. Instead, if PCP profit is convex in the exchange rate (condition (1) is not satisfied), the sign of $\Delta(S)$ depends on the value of the manager's absolute risk aversion relative to PCP profit convexity. Indeed, we then have $\Delta(S) > 0$ if and only if equation (4) is met:

$$-\frac{u''(.)}{u'(.)} > \frac{\pi''(.)}{(\pi'(.))^2}.$$

Note the separation theorem does not hold under PCP or HPCP. Indeed, risk aversion affects the optimal price because in contrast to the LCP case, exchange rate surprises affect demand under PCP and HPCP. Therefore, one cannot get a condition equivalent to (A.3). However, if prices could be set *after* the exchange rate were known, PCP and HPCP would yield the same profits: All variables are then known and the exporter can set p^{PCP} and p^{HPCP} optimally. Therefore, we have

$$u\left(\pi^{HPCP}(\mathbb{E}[S])\right) = u\left(\pi^{PCP}\left(\mathbb{E}[S]\right) - F\right),$$

so that similar to condition (2) in the case of LCP, the firm hedges as much

[2] The demonstration would then be similar to that above when firms choose between LCP and HLCP.

as it can under PCP if the condition (3) is satisfied:

$$u\left(\pi^{PCP}\left(\mathbb{E}\left[S\right]\right)\right)-\mathbb{E}\left[u\left(\pi^{PCP}(S)\right)\right]>\frac{du\left(\pi^{PCP}\left(\mathbb{E}\left[S\right]\right)\right)}{d\pi^{PCP}\left(\mathbb{E}\left[S\right]\right)}F+\Delta(S).$$

Depending on the sign of $\Delta(S)$, condition (3) is more or less stringent than (2). If condition (1) is met, (3) is more stringent than (2). Instead, if condition (1) is not met, (3) is more stringent than (2) only if (4) is also satisfied. Otherwise, it is less stringent. QED.

A.4 Invoicing conditional on hedging

The last stage to characterize the firm's joint decision of hedging and invoicing consists in comparing the HLCP and HPCP strategies, i.e. the decision of invoicing, conditional on hedging. From the firm's program, we can show that firm chooses HLCP over HPCP whenever:

$$\mathbb{E}\left[u\left(\pi^{HLCP}(S)\right)\right]-\mathbb{E}\left[u\left(\pi^{HPCP}(S)\right)\right]>0.$$

As shown in section A.3, we have:

$$\begin{aligned}\mathbb{E}\left[u\left(\pi^{HLCP}(S)\right)\right] &= u\left(\pi^{HLCP}(\mathbb{E}[S])\right). \\ &= \mathbb{E}\left[u\left(\pi^{LCP}(S)\right)\right]-\tfrac{du\left(\pi^{LCP}(\mathbb{E}[S])\right)}{d\pi^{LCP}(\mathbb{E}[S])}F\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\left[u\left(\pi^{HPCP}(S)\right)\right] &= u\left(\pi^{HPCP}(\mathbb{E}[S])\right)-\Delta(S) \\ &= \mathbb{E}\left[u\left(\pi^{PCP}(S)\right)\right]-\tfrac{du\left(\pi^{PCP}(\mathbb{E}[S])\right)}{d\pi^{PCP}(\mathbb{E}[S])}F-\Delta(S)\end{aligned}$$

From this, it comes that HLCP is preferred to HPCP if $\Delta(S)>0$ which, as discussed in Section A.3, happens if either condition (1) or condition (4) is met.

A.5 Extension to a more general hedging cost

On top of the fixed cost assumed in Section 3.2 of the paper, we could assume hedging costs entail a variable component. Although, in reality, the variable costs of hedging are likely decreasing in the amount hedged, we discuss in this appendix the robustness of our results to variable costs that are increasing in the amount hedged, which is the only situation that may eventually overturn some of the results in the text. We now explain why the qualitative results in

3.2 are not modified when we add a hedging cost component that increases in the quantity hedged h. Assume

$$HC[h] = c(h) + F,$$

where $c(h)$ is the variable cost component. With a variable cost component that is increasing in h (i.e., when $c'(h) > 0$), the optimal strategy no longer necessarily involves full hedging. Instead, the firm chooses h to maximize expected utility $\max_{p^i,h} \mathbb{E}\left[u\left(\pi^i(S)\right)\right]$. The first-order condition with respect to h is $\mathbb{E}\left[\frac{du\left(\pi^i(S)\right)}{d\pi^i(S)}\left(-S + f - c'(h)\right)\right] = 0$. This condition, together with $f = \mathbb{E}(S)$, implies $\text{Cov}\left[\frac{du\left(\pi^i(S)\right)}{d\pi^i(S)}, -S\right] = \mathbb{E}\left[\frac{du\left(\pi^i(S)\right)}{d\pi^i(S)}\right] c'(h)$, so that the exporter does not necessarily fully hedge when hedging costs entail a variable component. We are not able to determine the optimal quantity hedged without further assumptions on the relative curvature of the utility function and the hedging cost.

To highlight the fact that our qualitative results continue to hold, note $\frac{d^2u\left(\pi^i(S)\right)}{d(\pi^i(S))^2} < 0$ so that the term $\mathbb{E}\left[\frac{du(\pi^i(S))}{d\pi^i(S)}\right]$ is lower for larger (more profitable) firms. As long as the variable cost of hedging $c(h)$ is not too convex in the quantity hedged h, we have that $\text{Cov}\left[\frac{du\left(\pi^i(S)\right)}{d\pi^i(S)}, -S\right]$ is lower for larger firms. In words, our main result that larger firms are more likely to hedge therefore continues to hold except for extreme cases in which the variable cost is very convex in the quantity hedged. In the realistic case in which the variable cost is decreasing in h (i.e., $c(h)$ is concave in h), larger firms with larger hedging demand are even more likely to hedge than smaller firms, reinforcing our model's prediction.

The more risk averse the exporting firm's manager, the more likely the benefits from hedging are to outweigh the costs for larger firms. When hedging costs entail a variable cost component, this latter should not be too convex for hedging to be optimal.

B On the non-linear relationship between size and invoicing currency choice

The model highlights that introducing the option to hedge affects the fundamental determinants at the root of the invoicing decision. We now illustrate this point by focusing on a particular dimension of firms' invoicing strategies that has been extensively discussed in the previous literature; namely, the relationship between firm size and invoicing currency. We illustrate this point in three different contexts, a simple CES framework, an oligopolistic competition framework, and a framework in which risk aversion varies with firm size.

Let us consider first a standard CES model, with monopolistic competition, constant returns to scale, and no operational hedging. Using the notations introduced earlier, this implies $d \ln \eta / d \ln \frac{p^{PCP}}{S} = 0$, $mc_q = 0$ and $mc_S = 0$. Under these assumptions and without hedging, any firm would choose to price in its own currency as condition (1) cannot be met. The reason is that expected profits are always larger in PCP than in LCP. Such model is thus unable to explain the empirical relationship between firm size and invoicing currency choice. If one introduces the option to hedge instead, then, conditional on a homogenous degree of risk-aversion, the largest firms choose to hedge against exchange rate risk and price in the foreign currency, whereas small firms keep choosing PCP. The presence of hedging thus shifts the invoicing currency choice of the largest firms, which induces a correlation between firm size and invoicing currency choice.

We have discussed earlier the emergence of a non-linear relationship between firm size and invoicing currency choice in the context of oligopolistic competition (see the derivation in section A.2). Under constant returns to scale, no operational hedging, oligopolistic competition à la Atkeson & Burstein (2008), and in the absence of hedging, small and large firms choose PCP whereas medium-size firms choose LCP. The introduction of hedging in such a framework could however kill the hump-shaped relationship between size and invoicing currency. Indeed, if they are risk averse, the largest firms may find it optimal to hedge and price in the local currency (HLCP), despite such strategy reducing the level of expected profits.

The introduction of hedging in an oligopolistic competition model can thus overturn the non-linear relationship between size and invoicing currency that such models entail in the absence of hedging. Conversely, one can think of a model parametrization in which hedging can create the non-linearity which the empirical literature has documented. Namely, suppose that there is heterogeneity in risk aversion and large firms are less risk-averse.[3] For simplicity, start from a model in which all firms choose PCP without hedging. In such model, we might see in equilibrium both small and large firms pricing in PCP, while medium-size firms price in the importer's currency and subscribe hedging instruments. PCP decisions at the bottom and the top of the firm

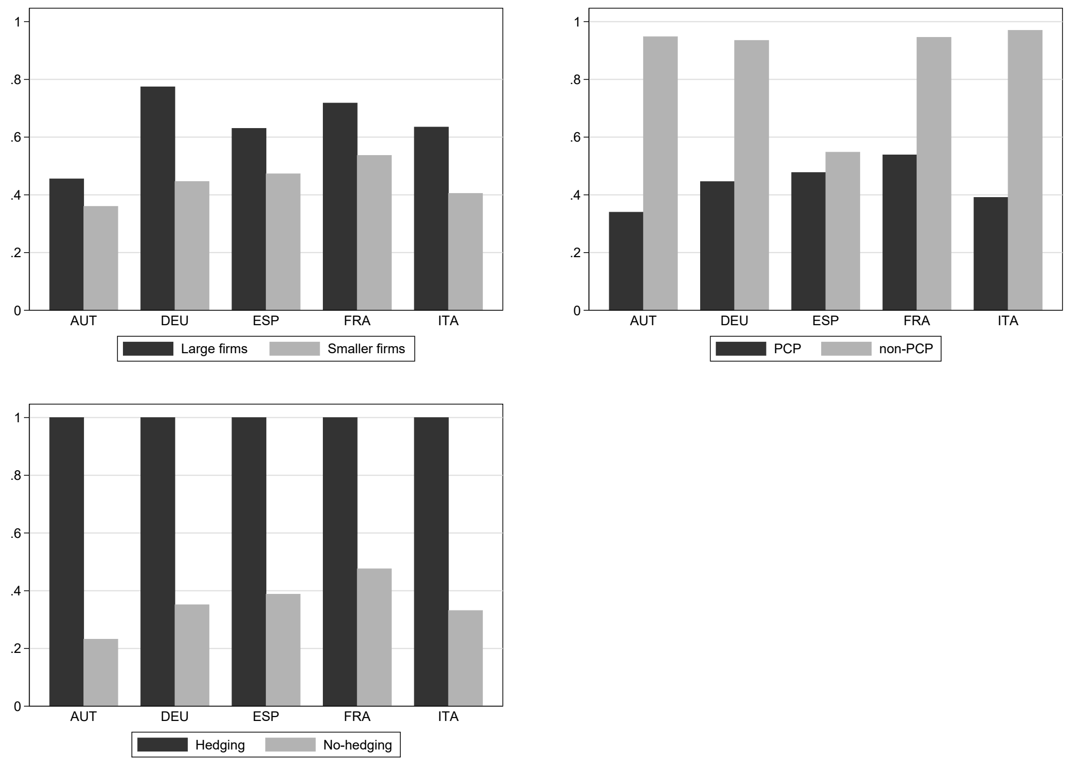
[3]The assumption that large firms are less risk averse is consistent with Froot et al. (1993). However, recent evidence suggest larger firms effectively behave as more risk averse because they own more collateral, which allows them to engage in risk management while maintaining their debt capacity (Rampini et al. 2020).

size distribution would then be due to small firms finding the fixed cost of hedging too large in comparison with the benefit, and large firms' low risk aversion would make them give preference to a higher expected profit. Instead, medium-size, risk-averse firms would choose the less uncertain HLCP strategy. The introduction of hedging together with heterogeneous risk aversion thus offers an alternative rationale for the hump-shaped relationship between firm size and ERPT uncovered by Auer & Schoenle (2016).

This discussion thus shows the relationship between firm size and invoicing currency choice depends on firms' option to hedge against exchange rate risk. For this reason, the sole observation of the impact of firm size on currency choice cannot be used to discriminate among models.

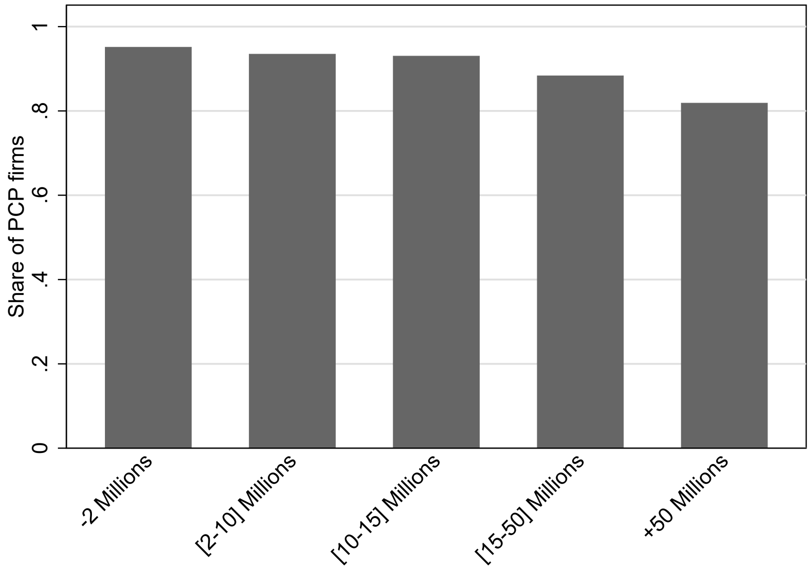
C Additional empirical results

Figure C.1: Heterogeneity in exchange rate exposure



Notes: This graph displays the share of firms from each country that are exposed to exchange rate risks in various populations of i) small and large firms, ii) PCP and non-PCP firms and iii) firms that are hedged against firms that are not. Large and small firms are defined as firms with sales above and below 50 millions euros.

Figure C.2: Heterogeneity in invoicing strategies across size bins



Notes: This graph displays the share of firms that declare pricing in their own currency, by size bins.

Table C.1: Determinants of currency choices: Baseline results over sub-samples of firms

	Dep.Var: Probability(PCP)				
	(1)	(2)	(3)	(4)	(5)
Sales above 50 millions	-0.48***	-0.54***	-0.46***	-0.49***	-0.53***
	(-4.841)	(-4.067)	(-3.485)	(-4.847)	(-5.048)
Share of exports	-0.54***	-0.61***	-0.51***	-0.53***	-0.51***
	(-3.856)	(-3.047)	(-3.303)	(-3.743)	(-3.539)
No Pricing Power	-0.21**	-0.23**	-0.22**	-0.20**	-0.19**
	(-2.537)	(-2.094)	(-2.395)	(-2.435)	(-2.329)
Multinational	-0.13	-0.08		-0.15	-0.16
	(-1.340)	(-0.612)		(-1.541)	(-1.623)
Sh. Oth. EU	0.00	0.00	0.00	0.00	0.00
	(0.673)	(0.752)	(0.281)	(0.562)	(0.517)
Sh. Other Eur.	-0.00	0.01	-0.00	-0.00	-0.00
	(-1.097)	(1.531)	(-1.276)	(-1.130)	(-1.186)
Sh. Chn-Ind	-0.01***	-0.01**	-0.01***	-0.01**	-0.01***
	(-3.049)	(-2.476)	(-2.748)	(-2.480)	(-2.608)
Sh. Other Asia	-0.01**	-0.01*	-0.01*	-0.01**	-0.01***
	(-2.529)	(-1.830)	(-1.831)	(-2.545)	(-2.600)
Sh. North Am.	-0.01***	-0.01***	-0.01***	-0.01***	-0.01***
	(-6.322)	(-3.652)	(-5.798)	(-6.160)	(-5.990)
Sh. South Am.	-0.01***	-0.01***	-0.02***	-0.01***	-0.01***
	(-5.970)	(-3.961)	(-5.741)	(-5.640)	(-5.314)
Sh. Row	-0.00*	-0.00	-0.00	-0.00*	-0.00*
	(-1.708)	(-0.399)	(-0.876)	(-1.787)	(-1.766)
Origin country FE	yes	yes	yes	yes	yes
Sector FE	yes	yes	yes	yes	yes
# Observations	3,011	1,437	2,496	2,929	2,876

Notes: This table presents the estimated coefficients of a probit model. The explained variable is the probability that the firm set prices in euros (PCP strategy). The specification is the same as in Table 2, column (4), reproduced in column (1). Column (2) is restricted to firms with at least one main partner outside of the EMU. Column (3) neglects firms which are part of a mutinational company. Column (4) neglects firms producing oil or metal products. Column (5) neglects firms in sectors in which at least 50% of firms declare that their price is fixed by the market. T-statistics computed from robust standard errors are reported in parenthesis. ***, ** and *, respectively, indicate significance at the 1, 5, and 10% levels.

References

Amiti, M., Itskhoki, O. & Konings, J. (2014), 'Importers, Exporters, and Exchange Rate Disconnect', *American Economic Review* **104**(7), 1942–78.

Atkeson, A. & Burstein, A. (2008), 'Pricing-to-Market, Trade Costs, and International Relative Prices', *American Economic Review* **98**(5), 1998–2031.

Auer, R. A. & Schoenle, R. S. (2016), 'Market structure and exchange rate pass-through', *Journal of International Economics* **98**(C), 60–77.

Burstein, A. & Gopinath, G. (2014), International prices and exchange rates, *in* G. Gopinath, E. Helpman & K. Rogoff, eds, 'Handbook of International Economics', Vol. 4 of *Handbook of International Economics*, Elsevier, chapter 7, pp. 391–451.

Froot, K. A., Scharfstein, D. S. & Stein, J. C. (1993), ' Risk Management: Coordinating Corporate Investment and Financing Policies', *Journal of Finance* **48**(5), 1629–58.

Rampini, A. A., Viswanathan, S. & Vuillemey, G. (2020), 'Risk management in financial institutions', *The Journal of Finance* **75**(2), 591–637.